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Abstract

We study the implications of assuming different energy intensities for physical capital accumulation and final good production in an overlapping generations (OLG) resource economy. Differing from the standard OLG literature, but consistently with the empirical evidence, physical capital accumulation is assumed to be relatively more energy-intensive than consumption. Focusing on exhaustible resources, we find that OLG equilibria can exhibit a "non-classical behaviour": our model can generate complex dynamics where extraction may increase during some periods and decrease afterwards. As a consequence, in contrast to the classical response predicted by the standard approach, resource prices may not increase monotonically. This result points out the importance of the assumptions about energy-intensity considered in the literature.

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1 Introduction

How does the scarcity of natural resources limit economic growth and to what extent physical capital accumulation offsets this constraint? These questions have been the research focus of many papers in the literature of resource economics dating back to Dasgupta and Heal (1974, 1979), Solow (1974), and Stiglitz (1974). In these studies, resources are assumed to be extracted in order to provide energy that will be used in the final good production. The limit to economic growth is then directly imposed by the usage of scarce resources in production. Greater physical capital accumulation is suggested (unless non-renewable resources are substituted with renewables) to overcome this constraint. However, the overwhelming majority of the literature, assuming the same technology for both consumption and physical capital accumulation, tends to contradict the empirical evidence about the energy intensity of these sectors. Data suggests that physical capital production is relatively more energy intensive than consumption, so that non-renewable resources could limit growth through the equipment production sector.[1] In this paper we analyze the effect of differentiating the energy intensities of physical capital and final good production in an overlapping generations (OLG) exhaustible resource economy. We study how the standard results regarding the dynamics of resources and prices are modified if physical capital accumulation is assumed to be relatively more energy-intensive than consumption.

To our knowledge, differing energy intensities has only been studied in Pérez-Barahona (2011), considering an infinitely lived agents general equilibrium model.

[1] Azomahou *et al.* (2004 and 2006) builds an energy intensity measurement (ratio between energy consumption and value added) of 14 sectors of the economy from the Structural Analysis Database of OECD and the Energy Balances and Energy Prices and Taxes of IEA. Their empirical analysis shows that energy intensity is high in sectors closely involved in production of physical capital (*e.g.*, iron and steel sector, 0.809; transport and storage, 0.85; non-ferrous metals, 0.599; and non-metallic minerals, 0.507). However, energy intensity is lower for consumption goods related sectors (*e.g.*, food and tobacco, 0.134; textile and leather, 0.082; and construction, 0.018). In the same direction, Williams (2004) points out that the "life cycle energy use of a computer is dominated by production (81%) as opposed to operation (19%)".

This paper shows that the monotonicity properties of consumption and resource extraction change if physical capital accumulation is assumed to be relatively more energy-intensive than consumption. The study is limited however to infinitely lived agents models, while numerous papers claim that the OLG framework offers a better explanatory power for the discussion of natural resource problems.[2] As in Pérez-Barahona (2011), we assume that the physical capital sector is more energy intensive than consumption, and energy is obtained from the extraction of natural resources. Thus, going beyond the standard literature, in our paper the accumulation of physical capital is assumed to be determined not only by the savings but also by the energy that it requires. In addition, instead of taking resources as the only way of saving (Krautkraemer and Batina, 1999; Koskela *et al.*, 2002), both resource stock and man-made capital are considered to be alternative assets (Mourmouras, 1991; Farmer, 2000; Agnani *et al.*, 2005; Bednar-Friedl and Farmer, 2013).

In the literature of natural resources and OLG models, the equilibrium trajectories converge to a single steady-state (or single balanced growth path) with saddle-path dynamics under linear/no regeneration of resources and with exogenous/no technological progress. The standard results on growth and dynamics in OLG exhaustible resource economies are mainly built upon the studies of Mourmouras (1991), and Olson and Knapp (1997). It is shown that with exhaustible resources, no technical progress, and standard assumptions about preferences and technology, extraction trajectories monotonically decrease (and therefore, resources price monotonically increases) into the trivial steady-state in the limit. Olson and Knapp (1997) find however that under non-standard assumptions (finite horizon quadratic utility), the convergence to the zero steady-state does not need to be monotone. Extraction could transitory rise, with the corresponding decrease of

[2] Three main reasons are frequently quoted. First, resources are a store of value between generations (see Koskela *et al.*, 2002; Valente, 2008; Bednar-Friedl and Farmer; 2013). Second, intergenerational aspects should be taken into account when analyzing environmental issues and/or natural resource economies (Solow, 1974; Padilla, 2002; and Agnani *et al.*, 2005). Finally, there exists empirical evidence showing that agents are not perfect altruistically linked (among others, Altonji *et al.*, 1992; and Balestra, 2003).

resources price. Moreover, cycle extraction is shown to be numerically possible under infinite horizon, CES preferences, and for a very particular set of parameters. Mourmouras (1991) studies the interaction between capital accumulation and natural resources exploitation under linear/no regeneration of resources. Besides from how physical capital accumulates, this model is similar to ours but the equilibrium dynamics are saddle-path stable.[3] Differing from this approach, our paper contributes to the literature by showing that, without taking linearizing assumptions, our model can generate complex dynamics and the subsequent non-monotonic behaviour of resources extraction and prices.

In absence of technological progress, we fully characterized the global dynamic response of our economy with respect to the share of exhaustible resources in the production of physical capital. The main contribution of our work comes from this characterization. In contrast to Mourmouras (1991), local indeterminacy and hopf bifurcations can arise if the share of energy resources in the equipment production is low enough. Moreover, a non-monotonic behavior of extraction is possible: the stock of (exhaustible) resources always decreases, yet extraction may increase during some periods and decrease afterwards. As a consequence, in contrast to the "classical behavior" predicted by the standard approach building on Hotelling's (1931) seminal work, resource prices may not increase in the short-run. Non-renewable resources literature widely discussed the discrepancy between the famous Hotelling prediction and the empirical evidence on resource prices. In fact, the prediction of monotonically rising prices has not been consistent with the empirical dynamic behavior of non-renewable resource prices and their in situ values.[4] Investigation of mineral commodity prices shows relative declines and fluctuation around time trends, rather than persistent increases, for long periods of time (see figures 1-11 in Krautkraemer, 1998). Considering a large data set of U.S. mineral prices, Barnett and Morse (1963) were the first to systematically identify this pattern. Slade (1982) further reveals that a U-shaped price evolution is consis-

[3]For a detailed analysis of the dynamics in Mourmouras (1991) see Farmer and Friedl (2010).
[4]See Krautkramer (1998) for a discussion of this literature.

tent with the observed prices of many non-renewable resources over the period 1870-1978 (for a recent study, see Ferraro and Peretto, 2003). Several modifications of Hotelling's model were then introduced in order to explain the behavior of prices over time. Environmental constraints and the natural resource abundance (Ahrens and Sharma, 1997), backstop technologies (Heal, 1976), technical change (Slade, 1982) and informational asymmetries (Pindyck, 1980) are possible known channels for inducing resources price decrease. In this regard, our paper identifies an additional channel for this "non-classical response" of prices, which is based on the energy intensity characteristics of the economy. We provide at the same time a justification for the emergence of hopf cycles and indeterminacy, *i.e.*, multiple equilibria. The interaction between energy intensity, discount rate and the share of physical capital in the final good production induces cyclic behavior in the economy via hopf bifurcations. Hopf bifurcations are economically important as they provide a powerful and easy tool to detect limit cycles and justify the emergence of cycles endogenously (for further details, see Benhabib and Farmer, 1999; Kind, 1999).[5]

Finally, another important feature of the paper is that our results do not rely on particular parameterizations of the exogenous functions involved in the model. It rather provides a flexible set-up to study the role played by the energy intensity, keeping the model tractable, together with general and plausible qualitative properties. The results are in fact analytical, and the dynamics are fully characterized. The paper proceeds as follows. The model is presented in Section 1. The competitive equilibrium is defined in Section 2. Section 3 presents the equilibrium dynamics and examines the stability of the long-run response. Conclusions and broader theoretical implications are discussed in Section 4.

[5]Hopf cycles appear when a fixed point loses or gains stability due to a change in a parameter and meanwhile a cycle either emerges from or collapses into the fixed point (Asea and Zak, 1999). Depending on the stability of the cycles a stylized business cycle (attracting/stable cycle) or corridor stability (repelling/unstable cycle) can appear (Kind, 1999; Benhabib and Miyao, 1981).

2 The model

We study a perfect foresight overlapping generations economy, without population growth, in discrete time with infinite horizon. In contrast to the standard OLG approach, our model differentiates the energy intensity of the physical capital sector and the corresponding of the final good production.[6] There are three sectors in the economy: final good production, equipment (investment) good production and extraction sector. A single final good, which can be either consumed or invested, is produced in the economy by combining physical capital and labor. The physical capital that is used in this production process is provided by the equipment sector. The physical capital is obtained by means of the already installed equipment and the energy supplied by the extraction sector. Finally, the extraction sector gets the energy directly from the natural resource extracted. All agents have rational expectations and each generation consists of a single representative agent. Moreover, they are price-takers and all the markets are assumed to be competitive.

2.1 Extraction sector

The purpose of natural resources in the model is double. They are both a store of value, as an asset, and an input in the production of investment good as energy. Following Agnani *et al.* (2005), we assume a grandfathering economy (due to the ownership of natural resources) so that the initially old generation possesses the stock of natural resources. At the beginning of each period t, the old agents own the resource stock $E_t = e_t N_t$, where e_t is the amount of resources per-worker and N_t is the number of workers. They choose how much to extract from the resource stock in order to sell it as energy to the equipment good sector, X_t. The remaining part of the natural resource is sold to the young agents as resource assets, $A_t (= E_t - X_t)$. From period t to $t+1$, the resource stock regenerates at a linear rate Π (Mourmouras, 1991), where $\Pi \geqslant 1$.[7] The transition dynamics of

[6] For the discussion of standard OLG models see de la Croix and Michel (2004).

[7] Note that the resource is non-renewable as long as $\Pi = 1$.

energy resources in per-worker terms can be formalized as follows:

$$e_{t+1} = \Pi(e_t - x_t) = \Pi a_t, \tag{1}$$

where $x_t \equiv X_t/N_t$ and $a_t \equiv A_t/N_t$.

2.2 Consumers

The representative individual receives an income equal to the real wage w_t from supplying her one unit of labor to the firms when young. She allocates her income among current consumption, c_t, savings related to investment, s_t, and the purchase of natural resource ownership rights, a_t. In her last period of life (when old at period $t+1$), the agent is retired and consumes d_{t+1} out of her entire income, not leaving any bequests. Her income is then generated from the return on her savings made when young, $R_{t+1}s_t$, from extracting the demanded portion of the energy resources and selling it to the firms, $Q_{t+1}x_{t+1}$, and from selling the rest to the young, $P_{t+1}a_{t+1}$, where R_t is the interest rate, and Q_t and P_t are the corresponding prices. Accordingly, the budget constraints facing generation t is as follows:

$$c_t + s_t + P_t a_t = w_t \ , \tag{2}$$

$$d_{t+1} = R_{t+1}s_t + Q_{t+1}x_{t+1} + P_{t+1}a_{t+1}. \tag{3}$$

Generations derive utility from consumption, where their two-period intertemporal utility function dependens on the level of consumption when young c_t and when old d_{t+1}. We assume an additively separable life-cycle utility function $U(c_t, d_{t+1}) = u(c_t) + \beta u(d_{t+1})$, where $\beta \in (0,1)$ is the subjective discount factor. In particular, we adopt a logarithmic instantaneous utility function u since we are mainly concerned with the existence of the competitive equilibrium and its qualitative properties.

Taking the prices of the energy resource and wages as given, the representative agent born at time t maximizes her utility by choosing young and old periods' consumption, and the ownership rights of the energy resource. The corresponding optimization problem of the representative consumer is formalized as follows:

$$\max_{\{c_t, d_{t+1}, s_t, e_{t+1}\}} \ln c_t + \beta \ln d_{t+1}$$

subject to

$$\begin{aligned}
c_t + s_t + P_t a_t &= w_t \ , \\
d_{t+1} &= R_{t+1} s_t + Q_{t+1} x_{t+1} + P_{t+1} a_{t+1}, \\
e_{t+1} &= \Pi(e_t - x_t) = a_t \\
c_t &\geq 0, d_{t+1} \geq 0, e_{t+1} \geq 0, E_0 \ > 0 \text{ given.}
\end{aligned}$$

We get the following first-order conditions:

$$\frac{d_{t+1}}{c_t} = \beta R_{t+1}, \tag{4}$$

$$\frac{P_{t+1}}{P_t} = \frac{R_{t+1}}{\Pi}, \tag{5}$$

$$P_{t+1} = Q_{t+1}. \tag{6}$$

Equation (4) gives the equalization of discounted marginal utilities, where the marginal rate of substitution between current and future consumption is equal to their relative prices. Equation (5) is the non-arbitrage condition between the different types of savings (savings related to investment and ownership rights). Finally equation (6) is a non-arbitrage condition, which implies that in the equilibrium the asset price and the extracted energy price are the same. This last result is indeed consistent with Olson and Knapp (1997) and Valente (2008). Moreover, notice that equations (5) and (6) are equivalent to the Hotelling rule presented in Mourmouras (1991).

2.3 Firms

2.3.1 Final good sector

Firms operating in the final good sector are owned by the old households. Firms produce the final good with a Cobb-Douglas constant returns to scale technology. Equation (7) presents this production function at any date t. The exogenous disembodied total factor productivity is represented by Z_t (equation 8). With constant returns to scale, the number of firms does not matter and the production is independent of the number of firms that use the same technology. We are so concerned with the problem of a representative firm. Under this perfectly competitive environment, at each period t, taking the prices of inputs, the initial technology level, and the initial level of capital stock as given, the representative firm maximizes its profit by choosing the amount of labor and physical capital (equipment) inputs:

$$\max_{\{K_t,N_t\}_{t=0}^{\infty}} \pi_t = Y_t - P_t^K K_t - w_t N_t,$$

$$s.t. \quad Y_t = Z_t K_t^{\alpha} N_t^{1-\alpha} \qquad 0 < \alpha < 1, \tag{7}$$

$$Z_{t+1} = (1+z)Z_t \qquad z \geq 0. \tag{8}$$

At an interior solution of the firm's optimization problem, where all variables are expressed in per-worker terms ($k_t \equiv K_t/N_t$), the following first-order conditions are satisfied equating the price of the inputs to their marginal benefits:

$$\alpha y_t = P_t^K k_t, \tag{9}$$

$$(1-\alpha)y_t = w_t. \tag{10}$$

Equation (11) summarizes the market clearing condition of the economy. The final good is either consumed by young agents, C_t, by old agents (generation $t-1$), D_t, or invested for the production of the future capital stock, S_t:

$$Y_t = C_t + D_t + S_t. \tag{11}$$

2.3.2 Physical capital sector

In the standard OLG literature, the new capital stock at time $t+1$ is fully determined by the savings made at time t, which are equal to the investments. However, following Pérez-Barahona (2011), since the physical capital production is relatively more energy-intensive than consumption we model the accumulation of capital stock to be determined not only by the savings made at time t, but also by the energy that it requires.[8] The usage of natural resource introduces then a new constraint on economic growth through the capital accumulation sector. In equation (12) the new capital at $t+1$, K_{t+1}, is produced from the natural energy resources X_{t+1} and the investment made at time t, I_t , with the following Cobb-Douglas technology:

$$\begin{aligned} K_{t+1} &= B_{t+1}^{\theta} X_{t+1}^{\theta} I_t^{1-\theta} \qquad 0<\theta<1, && (12) \\ S_t &= I_t, && (13) \\ B_{i+1} &= (1+b)B_t, \qquad\qquad b \geq 0. && (14) \end{aligned}$$

It is worthwhile to mention that the savings S_t are still equal to the investments (equation 13). However, only a fraction of the investments can generate the new capital stock. B_t is the technological progress in the equipment good sector. In contrast to Z_t, B_t is energy-saving and specific to the accumulation of physical capital. If B_t increases, the productivity of the natural resources in the production of equipment rises. Consequently, a lower amount of energy would be necessary to produce the same amount of new physical capital. Changes in B_t also represent investment-specific technological change, which is assumed to affect exclusively the equipment sector.

In the equipment sector, at each period t, the representative firm maximizes its profit by choosing the amount of energy resources that will be used in the production process:

$$\begin{aligned} \max_{\{X_t\}_{t=0}^{\infty}} \pi_t &= P_t^K K_t - Q_t X_t, \\ s.t. \quad K_t &= B_t^{\theta} X_t^{\theta} I_{t-1}^{1-\theta}, \end{aligned}$$

taking as given the prices of capital (P_t^K) and resource input (Q_t), and the initial level of capital stock. At an interior solution of the equipment firm's optimization

[8] We should observe that in our economy energy is not a direct input to produce the final good. As in Pérez-Barahona (2011), this simplification is considered in order to highlight the importance of the energy intensity differences between consumption and equipment sectors.

problem the following first-order condition is satisfied:

$$Q_t = \theta \alpha Y_t X_t^{-1}. \tag{15}$$

The profit of investing in capital $R_t S_{t-1}$ at time t should be equal to the profit of producing new capital $(1-\theta)P_t^K K_t$ to prevent arbitrage opportunities.[9] We then obtain the following condition:

$$R_t = (1-\theta)P_t^K B_t^{\frac{\theta}{1-\theta}} X_t^{\frac{\theta}{1-\theta}} K_t^{\frac{\theta}{\theta-1}}. \tag{16}$$

2.4 The competitive equilibrium

The dynamic competitive equilibrium for this OLG natural resource economy is defined as a sequence of prices $\{w_t, R_t, P_t^K, P_t, Q_t\}_{t=0}^{\infty}$ and feasible allocations $\{c_t, d_t, s_t, e_t, a_t, x_t, i_t, y_t, k_{t+1}\}_{t=0}^{\infty}$, given the positive initial values for S_{-1}, E_0, Z_0, B_0, N_0, and the law of motion of exogenous technological progresses Z_t and B_t, such that the consumers maximize their life-time utility, firms maximize their profits, and all markets clear at every period t. This equilibrium is therefore a solution of the system of equations (1)-(16).

3 Equilibrium dynamics

The main focus of our paper is to study the dynamics around a steady-state equilibrium. We therefore assume no technological progress, *i.e.*, $b = z = 0$. As in Mourmouras (1991), Farmer (2000) and Bednar-Friedl and Farmer (2013), the intertemporal equilibrium dynamics can be reduced to a two-dimensional system, which represents the law of motion of the resource stock, E_t, and the extraction, X_t. Using the equipment technology (12), the condition (13), the budget constraints

[9] Choosing the optimal resource stock (equation 15), the maximum profit that the representative firm can obtain is given by

$$\pi_t^* = P_t^K K_t - \theta P_t^K B_t^{\theta} X_t^{\theta-1} I_{t-1}^{1-\theta} X_t = (1-\theta)P_t^K K_t.$$

and first-order conditions (2)-(5) of the households' problem, and the first-order conditions (10) of the firms' maximization problem, we obtain the equation below for k_{t+1}:

$$k_{t+1}^{\frac{1}{1-\theta}} = B_{t+1}^{\frac{\theta}{1-\theta}} x_{t+1}^{\frac{\theta}{1-\theta}} \left[\sigma(1-\alpha) + \theta\alpha(1-\frac{e_t}{x_t})\right] y_t, \tag{17}$$

with $\sigma \equiv \beta/(1+\beta)$. In addition, taking equations (5), (6), (9), (15) and (16), the following difference equation for k_{t+1} arises:

$$\frac{\Pi k_{t+1}^{\frac{1}{1-\theta}} x_t}{y_t x_{t+1}} = \alpha(1-\theta) B_{t+1}^{\frac{\theta}{1-\theta}} x_{t+1}^{\frac{\theta}{1-\theta}}. \tag{18}$$

Substituting for k_{t+1} in the above equations yields (19) below, which describes the dynamics of resource extraction. Since it also depends on the resource stock e_t, we rewrite (1), the law of motion of the natural resource, as (20).

$$x_{t+1} = \Pi\left(\frac{\frac{\beta}{(1+\beta)}(1-\alpha) + \alpha\theta}{\alpha(1-\theta)}\right) x_t - \frac{\Pi\theta}{(1-\theta)} e_t, \tag{19}$$

$$e_{t+1} = \Pi(e_t - x_t). \tag{20}$$

The linear planar system (19) and (20) describes the dynamics of our economy, which can be rewritten in matrix form as

$$\begin{aligned}
\begin{bmatrix} x_{t+1} \\ e_{t+1} \end{bmatrix} &= \begin{bmatrix} \psi_1 & \psi_2 \\ \psi_3 & \psi_4 \end{bmatrix} \begin{bmatrix} x_t \\ e_t \end{bmatrix} \equiv \Psi \begin{bmatrix} x_t \\ e_t \end{bmatrix} \\
\textit{with } \psi_1 &\equiv \frac{\Pi}{(1-\theta)} \frac{\beta}{(1+\beta)} \frac{(1-\alpha)}{\alpha} + \frac{\Pi\theta}{(1-\theta)}, \\
\psi_2 &\equiv -\frac{\Pi\theta}{(1-\theta)}, \\
\psi_3 &\equiv -\Pi, \\
\psi_4 &\equiv \Pi.
\end{aligned}$$

Lemma 1 *(steady-states) The steady-state equilibria (x^*, e^*) of our economy are characterized by the following equations:*

$$x^* = \psi_1 x^* + \psi_2 e^*, \tag{21}$$

$$e^* = \psi_3 x^* + \psi_4 e^*. \tag{22}$$

These equations have two sets of steady-states:

(i) *If* $\{\psi_1 = 1\}$ *or* $\{\psi_1 \neq 1$ *and* $\psi_4 = 1\}$ *or* $\{\psi_1 \neq 1, \psi_4 \neq 1$ *and* $(1-\psi_4) \neq \frac{\psi_2\psi_3}{(1-\psi_1)}\}$ *there is a unique steady-state* $(x^*, e^*) = (0, 0)$.

(ii) *If* $\{\psi_1 \neq 1, \psi_4 \neq 1$ *and* $(1-\psi_4) = \frac{\psi_2\psi_3}{(1-\psi_1)}\}$ *there is a continuum of steady-states such that* $x^* = \frac{\psi_2}{1-\psi_1}e^*$.

Proof. A steady-state (x^*, e^*) in this economy is a fixed point of (19) and (20), where $x_{t+1} = x_t = x^*$ and $e_{t+1} = e_t = e^*$. Since $\Pi \geq 1$, $\psi_1, \psi_4 > 0$, and $\psi_2, \psi_3 < 0$, the sets (i) and (ii) are easily determined from (21) and (22) $\blacksquare$

From the linear system (21) and (22), one can verify that there is either a unique steady-state $(x^*, e^*) = (0, 0)$, or a continuum of them where natural resources are not necessarily drained. If resources are non-renewable ($\Pi = 1$) the only possible steady-state, without technical progress, is $(x^*, e^*) = (0, 0)$ because $\psi_4 = 1$ (set i). However, the natural replenishing property of renewable resources ($\Pi > 1$, thus, $\psi_4 \neq 1$) allows the economy to avoid depletion (set ii).

3.1 Stability and exhaustible resources

As pointed out in the introduction, the objective of this work is to study the dynamic implications of the energy intensity assumptions of the economy. In particular, our intention is to provide an alternative explanation for the non-classical behavior of exhaustible resources, which was empirically identified in the literature. Even if Section 2 offers a general framework for both exhaustible and renewable resources, from now on, we will concentrate on non-renewable resources and the dynamics around the corresponding unique steady-state $(x^*, e^*) = (0, 0)$. We analyze below the stability of the system, the occurrence of indeterminacy and bifurcations, and the subsequent possibility of non-monotone convergence. In this regard, we find out that the share of energy in the equipment production plays a fundamental role, together with the discount rate and the share of physical capital in the final good production. Notice finally that our conclusions will be global since the dynamics of the economy are described by a linear system.[10]

We denote by λ_1 and λ_2 the eigenvalues of Ψ in the matrix form of the dynamical system (19) and (20). Let us define $\tilde{\beta} \equiv \frac{\beta}{(1+\beta)} \frac{(1-\alpha)}{\alpha} > 0$. The next proposition fully describes the dynamics of our economy by means of studying λ_1 and λ_2. In particular, we identify cases where complex dynamics arise, allowing the economy to reproduce the non-classical response described before. These conditions are indeed deeply related to the assumption of considering that the equipment sector is more energy intensive than the final good production.

Proposition 1 *(stability) For non-renewable resources and different parameter combinations, the stability of the zero steady-state changes such that:*

1. *For* $\frac{1}{2} \leq \theta < 1$, *the dynamics are non-complex and the eigenvalues are on the different side of one* ($\lambda_1 > 1$ *and* $0 < \lambda_2 < 1$) *so that the steady-state is a saddle.*

2. *For* $0 < \theta < \frac{1}{2}$ *and* $2(1-2\theta) \leq \tilde{\beta}$, *the dynamics are non-complex and the eigenvalues are on the different side of one* ($\lambda_1 > 1$ *and* $0 < \lambda_2 < 1$) *so that the steady-state is a saddle.*

3. *For* $\frac{1}{4} \leq \theta < \frac{1}{2}$ *and* $\tilde{\beta} < 2(1-2\theta)$, *the dynamics are non-complex and the eigenvalues are on the different side of one* ($\lambda_1 > 1$ *and* $0 < \lambda_2 < 1$) *so that the steady-state is a saddle.*

4. *For* $0 < \theta < \frac{1}{4}$, $\tilde{\beta} < 2(1-2\theta)$ *and* $\tilde{\beta} + \frac{1}{\tilde{\beta}} > 2(1-2\theta)$, *the dynamics are non-complex and the eigenvalues are on the different side of one* ($\lambda_1 > 1$ *and* $0 < \lambda_2 < 1$) *so that the steady-state is a saddle.*

5. *For* $0 < \theta < \frac{1}{4}$ *and* $\tilde{\beta} + \frac{1}{\tilde{\beta}} < 2(1-2\theta)$ *the dynamics are complex:*

[10] The results of this paper correspond to global dynamics: they are valid regardless how far away the economy is from the steady-state. Therefore, in contrast to linearization and local dynamics, we provide a more general outcome.

(a) If $\theta < 1-\tilde{\beta}$ both eigenvalues (in absolute value) are smaller than one, so indeterminacy occurs and the steady-state is stable.

(b) If $\theta > 1-\tilde{\beta}$ both eigenvalues (in absolute value) are greater than one, so the equilibrium dynamics are monotone unstable.

Proof. For exhaustible resources $\Pi = 1$. If $(1+\psi_1)^2 \geq 4(\psi_1+\psi_2)$ the two eigenvalues of Ψ are real and given by $\lambda_{1,2} = \frac{(1+\psi_1)}{2} \pm \frac{1}{2}\sqrt{(1+\psi_1)^2 - 4(\psi_1+\psi_2)}$. Moreover, $\lambda_{1,2} > 0$ since the $\det(\Psi) > 0$ and $\mathrm{Tr}(\Psi) > 1$.[11] This implies that non-monotonic dynamics is only possible if the eigenvalues are complex: if $(1+\psi_1)^2 < 4(\psi_1+\psi_2)$, $\lambda_{1,2} = \frac{(1+\psi_1)}{2} \pm i\frac{1}{2}\sqrt{4(\psi_1+\psi_2)-(1+\psi_1)^2}$. We identify the cases of complex dynamics in the Appendix. Let us directly consider the stability analysis:

1. For $\frac{1}{2} \leq \theta < 1$, the dynamics are non-complex as the discriminant $\Delta \equiv (1+\psi_1)^2 - 4(\psi_1+\psi_2) > 0$ (see Claim 2 in Appendix). Since $\lambda_{1,2} > 0$, comparing λ_1 and λ_2 with 1, one can observe that the dynamics are stable iff both eigenvalues are smaller than one. This is equivalent to $\frac{(1+\psi_1)}{2} < 1 - \frac{\sqrt{\Delta}}{2}$. This is however impossible because $\frac{(1+\psi_1)}{2} > 1 - \frac{\sqrt{\Delta}}{2}$ (see Claim 3 in Appendix). Therefore, stability cannot occur.

   Let us study the possibility of saddle equilibrium or monotone unstable dynamics. The steady-state is saddle iff the two eigenvalues are on the different side of one. As $\lambda_{1,2} > 0$, this condition is equivalent to $\lambda_1 > 1$ and $\lambda_2 < 1$ (notice that, from Claim 1, $\lambda_1 > \lambda_2$). Taking the expression of $\lambda_{1,2}$, the condition reduces to $1 - \frac{\sqrt{\Delta}}{2} < \frac{(1+\psi_1)}{2} < 1 + \frac{\sqrt{\Delta}}{2}$. In contrast to that, the equilibrium dynamics are monotone unstable iff $\lambda_1 > 1$ and $\lambda_2 \geq 1$. This is equivalent to check that $\frac{(1+\psi_1)}{2} \geq 1 + \frac{\sqrt{\Delta}}{2}$. Since the dynamics are non-complex for $\frac{1}{2} \leq \theta < 1$, we can conclude from Claim 3 that $\frac{(1+\psi_1)}{2} < 1 + \frac{\sqrt{\Delta}}{2}$. We have also proved in Claim 3 that $\frac{(1+\psi_1)}{2} > 1 - \frac{\sqrt{\Delta}}{2}$. Therefore, $1 - \frac{\sqrt{\Delta}}{2} < \frac{(1+\psi_1)}{2} < 1 + \frac{\sqrt{\Delta}}{2}$ and, consequently, the steady-state is a saddle.

[11]Notice that, $\lambda_1\lambda_2 = \det(\Psi) = \psi_1 + \psi_2 > 0$. Since $\det(\Psi) > 0$, $\lambda_{1,2} \neq 0$ and their sign coincide. Moreover, $\lambda_1 + \lambda_2 = \mathrm{Tr}(\Psi) = 1 + \psi_1 > 1$. We can then conclude that both eigenvalues are strictly positive.

2. For $0 < \theta < \frac{1}{2}$ and $\tilde{\beta} \geq 2(1-2\theta)$ the dynamics are non-complex (Claim 2). Therefore, from Claim 3, $\frac{(1+\psi_1)}{2} < 1 + \frac{\sqrt{\Delta}}{2}$. From the same claim we also know that, if $0 < \theta < \frac{1}{2}$ and $\tilde{\beta} \geq 2(1-2\theta)$, $\frac{(1+\psi_1)}{2} > 1 - \frac{\sqrt{\Delta}}{2}$. We can then conclude that $1 - \frac{\sqrt{\Delta}}{2} < \frac{(1+\psi_1)}{2} < 1 + \frac{\sqrt{\Delta}}{2}$, thus the steady-state is a saddle.

3. We know from Claim 3 that, if $\frac{1}{4} \leq \theta < \frac{1}{2}$ and $\tilde{\beta} < 2(1-2\theta)$, $1 - \frac{\sqrt{\Delta}}{2} < \frac{(1+\psi_1)}{2}$. Moreover, $\frac{(1+\psi_1)}{2} < 1 + \frac{\sqrt{\Delta}}{2}$ because we are in a case on non-complex dynamics (see Claim 3). Therefore, as above, the steady-state is a saddle.

4. It is easy to very that statements 5 and 6 in Claim 3 are also valid for $0 < \theta < \frac{1}{4}$ and $\tilde{\beta} < 2(1-2\theta)$ if the dynamics are non-complex. Following Claim 2, we are in a case of non-complex dynamics. We can then conclude that $1 - \frac{\sqrt{\Delta}}{2} < \frac{(1+\psi_1)}{2} < 1 + \frac{\sqrt{\Delta}}{2}$, thus the steady-state is a saddle.

5. From Claim 2, we know that this is the case of complex dynamics. Taking the formulas for $\lambda_{1,2}$ above, one can verify that $|\lambda_1| = |\lambda_2| = \sqrt{\frac{\tilde{\beta}}{(1-\theta)}}$. Comparing $|\lambda_1| = |\lambda_2|$ with 1, we can directly conclude that the dynamics are stable iff the modulus of both eigenvalues is smaller than one. This condition is in fact equivalent to $\tilde{\beta} < (1-\theta)$. Similarly, the dynamics are unstable iff the modulus of both eigenvalues is larger than one, which corresponds to $\tilde{\beta} > (1-\theta)$.

■

The proposition clearly shows that energy intensity differences among economic sectors has important dynamic implications. In contrast to the standard approach, where a monotone converge of resource stock and extraction is predicted, our economy can also exhibit complex dynamics. This outcome is deeply related to the assumption that the equipment sector is more energy intensive than the final good production. Our property of non-monotone converge actually provides an alternative explanation to the empirically supported fact of the non-classical behaviour of exhaustible resources: resource stock and extraction can transitory increase (thus energy prices can temporally decrease) before converging to the zero steady-state.

Proposition 1 points out the role played by the share of resources in the accumulation of physical capital, θ. When the share of energy in the production of equipment is high ($\frac{1}{2} \leq \theta < 1$), we get the usual saddle-path dynamics. However, for a lower share ($0 < \theta < \frac{1}{2}$) the economy can reproduce situations of complex dynamics and, in particular, non-monotone convergence. In this respect, two additional elements become fundamental through the parameter $\tilde{\beta}$: the share of physical capital in the final good production (α) on the one hand, and the individuals' discount rate (β) on the other. For combinations of (α, β) such that $\tilde{\beta}$ would be low enough (*i.e.*, lower than $2(1-2\theta)$), complex dynamics arise. In this case, an additional upper-bound for θ appears in order to ensure the stability of the steady-state. This is statement 5 (a) in the proposition, which corresponds to the non-classical response of exhaustible resources. Since energy resources are non-renewable in this economy, the variable stock of resources e_t in the system (19)-(20) cannot increase. However, differing from non-complex dynamics, the extraction of resources (variable x_t) can increase during some periods, eventually converging (asymptotically) to the steady-state. Replacing Y_t in (15) by (7), and K_t by (12) afterwards, we get the usual negative relationship between extraction and energy prices Q_t. Therefore, energy prices can transitory decrease, rising later on as extraction approaches to the steady-state. This possibility of non-monotonic convergence is an important property of our framework. It highlights in fact the influence of the energy intensity assumptions on the dynamic predictions of the model.

The parameters described above are related to the importance that each generation gives to energy resources. In particular, they affect the value of the ownership rights for the natural resources a_t, which are sold from one generation to the next. In economies where θ is small (lower than 1/4 in our model), energy resources are relatively unimportant with respect to the other inputs in the equipment technology. Therefore, future generations give little value to the ownership rights of resources. The current generation would then not have many incentives to leave resources to the next generation, increasing thus the extraction when old. This will continue until energy becomes relative scarce. At this moment, the value of the ownership rights raises, creating new incentives to reduce extraction and, consequently, increasing the amount of resources left to the next generation.[12]

We should observe that the non-classical behaviour also depends on the share of equipment in the technology of final good and the discount rate through the parameter $\tilde{\beta}$. Proposition 1 (statement 5) identifies boundaries for $\tilde{\beta}$. The condition in (a) can be rewritten as $\tilde{\beta} < 1 - \theta$, providing an upper limit. Moreover, it also implies that $\tilde{\beta} < 2(1 - 2\theta)$ since $(1 - \theta) < 2(1 - 2\theta)$ for $0 < \theta < 1/4$. The second condition $\tilde{\beta} + 1/\tilde{\beta} < 2(1 - 2\theta)$ concludes then that $\tilde{\beta}$ cannot be too small either. Let us consider a reference economy for $0 < \theta < 1/4$ where α and β are such the conditions of statement 5 (a), and so the boundaries for $\tilde{\beta}$, hold. For a higher discount rate β, the parameter $\tilde{\beta}$ will increase and, therefore, the upper limit may not be respected anymore.[13] However, this effect can be counterbalanced if the economy has a higher share of physical capital in the technology of final goods (α). The interpretation is the following. A higher discount rate implies greater concern of consumers for their old age and, consequently, for the amount of resources they are going to extract or leave (sell) to the next generation. Since energy is not very important in this economy ($0 < \theta < 1/4$), the next generation will not value so much the ownership rights. Therefore, as above, the current generation may increase the resource extraction for some periods. Nevertheless, a larger share of equipment in final good technology would increase how much future generations value energy resources and the corresponding ownership rights, ensuring then that extraction will eventually decreases.

Notice finally that the upper bound for $\tilde{\beta}$ allow us to identify limit cycles. We show in the next corollary that the critical value $(1-\theta)$ reveals indeed the existence of a hopf-type bifurcation. Hopf cycles appear when a steady-state loses or gains stability due to a change in a parameter, and meanwhile a cycle either emerges

[12]Since consumers are non-altruistic and live for two periods, ownership rights prevent the old generation from extracting all resources.

[13]From the definition of $\tilde{\beta}$, it is easy to see that $\partial\tilde{\beta}/\partial\beta > 0$ and $\partial\tilde{\beta}/\partial\alpha < 0$.

from or collapses into the steady-state (for further details, see among others, Asea and Zak, 1999; Kind, 1999; and Yüksel, 2011).

Corollary 1 *(hopf bifurcation) Assume that* $0 < \theta < \frac{1}{4}$ *and* $\tilde{\beta} + \frac{1}{\tilde{\beta}} < 2(1 - 2\theta)$. *Indeterminacy occurs for* $\tilde{\beta} < 1 - \theta$*; a hopf bifurcation arises for* $\tilde{\beta} = 1 - \theta$*; and the steady-state is unstable for* $\tilde{\beta} > 1 - \theta$.

For small values of the share of energy in the equipment production, depending on its interaction with the capital share in the final good technology and the discount rate, the equilibrium trajectories could be indeterminate, implying the possibility of multiple equilibria (see Benhabib and Gali, 1995, for a survey). If $\tilde{\beta} < 1 - \theta$, in contrast to saddle stability, there would be multiple paths tending to the (unique) steady-state since many values of $x_0 \geq 0$ would be compatible with this equilibrium. Moreover, as showed before, the convergence in this case will be non-monotonic. If $\tilde{\beta} > 1 - \theta$ cycles arise, but the economy will not converge to the steady-state. These unstable dynamics can be interpreted considering again the importance that each generation gives to energy resources. Assume an economy with (α, β) such that it converges, non-monotonically, to the steady-state. If we sufficiently increase β and reduce α, the subsequent larger $\tilde{\beta}$ may induce cycles that will not converge to the steady-state (if $\tilde{\beta}$ is greater than $1 - \theta$). As before, a greater discount rate (β) implies that generations are more concerned about their old age and, therefore, about how much they will extract or leave to the next generation. If at the same time the importance of equipment input in the final good production (α) lowers, energy would not be central either. Therefore, the next generation will not value natural resources much, reducing the incentives of the old generation to replace extraction by ownership rights. Let us lastly mention that since resources are non-renewable in this paper, the non-monotonic trajectories corresponding to $\tilde{\beta} \geq 1 - \theta$ will end up in the steady-state but in a finite time, *i.e.*, non-asymptotically. Because the quantity of resources cannot increase, the current stock e_t is an upper-limit to how much the economy can extract in that period. Since the stock of resources always shrinks with extraction, the economy will completely deplete the resources in a finite time if $\tilde{\beta} \geq 1 - \theta$.

The zero steady-state then emerges, but as a corner solution.[14]

4 Concluding remarks

Although the literature widely assumes the same technology for consumption and capital accumulation sector, data suggest that physical capital production is relatively more energy-intensive than consumption and, thus, the usage of non-renewable resources can limit growth through the equipment production. Using an overlapping generations (OLG) natural resources model, we examine the dynamic implications of differentiating the energy intensities of physical capital sector and final good production. The model assumes that the equipment sector is more energy intensive than consumption, where energy is obtained from the extraction of natural resources. In contrast to the standard approach, we model the accumulation of physical capital to be determined not only by the savings but also by the energy that it requires.

We find that the introduction of energy-intensity differentiation among sectors has important implications for the standard results in the area. Richer dynamics other than saddle arise. In particular, energy extraction and prices can follow a non-monotonic trend that is consistent with the empirical finding about the non-classical behaviour of exhaustible resources. From an economic policy perspective, we believe that this paper provides a meaningful contribution too. On the one hand, we have identified a new component that improves our understanding about the puzzling issue of non-monotonic response of exhaustible resources. Our work makes compatible the theoretical predictions with the empirics in this regard. On the other hand, since proper policymaking depends on economic forecasts, "appropriately modeling the nature of the time series can be invaluable to forecasters" (Lee *et al.*, 2006). In this line, a general message of our paper is that the assumptions about the energy-intensity differences among sectors should be taken with

[14] Notice that stable (unstable) cycles can be sustained, for $\tilde{\beta} = (>) 1 - \theta$, if the stock of resources were not subject to the positivity constrain $e_t \geq 0$.

caution in the design of economic policies.

Several issues can be considered in future research. As pointed out before, our framework can be also applied to renewable resources by considering $\Pi > 1$. This would potentially create even richer dynamics. In particular, convergence to steady-states without exhaustibility may be possible since resources naturally reproduce. Extraction, moreover, would not necessarily induce lower future stock of resources. This could allow then, in contrast to the corner solutions observed before, the emergence of the long-run cycles. Following the same direction, another interesting hypothesis to examine is the role played by the regeneration law of natural resources. One could indeed consider alternative regeneration specifications to the usual linear replenishing of resources. In this respect, a logistic reproduction law has been often recommended to study forestry problems (Farmer, 2000; and Bednar-Friedl and Farmer, 2013, among others).

5 Appendix

5.1 Proof of Proposition 1

Claim 1 *For non-renewable resources, the discriminant* $\Delta \equiv (1+\psi_1)^2 - 4(\psi_1+\psi_2)$ *cannot be zero. In fact,*

$$\Delta > (<) 0 \Leftrightarrow 2\,(1-2\theta) - \tilde{\beta} < (>) \frac{1}{\tilde{\beta}}. \tag{23}$$

Moreover, for real eigenvalues, $\lambda_1 > \lambda_2$.

Proof. Let us prove that $\Delta \neq 0$. By contradiction, we assume that $\Delta = 0$. Therefore $(1+\psi_1)^2 = 4(\psi_1+\psi_2)$. Since $\psi_1 = \frac{\tilde{\beta}+\theta}{1-\theta}$ and $\psi_2 = -\frac{\theta}{(1-\theta)}$, the previous condition is equivalent to $\left(\frac{1+\tilde{\beta}}{1-\theta}\right)^2 = \frac{4\tilde{\beta}}{1-\theta}$. This is only possible iff $\tilde{\beta} = (1-2\theta) \pm \sqrt{(1-2\theta)^2 - 1}$. Since $(1-2\theta)^2 < 1$, $\tilde{\beta}$ would be a complex number. This contradicts however our definition of $\tilde{\beta}$, which is a real number. We indeed conclude that $\Delta > (<) 0$ iff $2\,(1-2\theta) - \tilde{\beta} < (>) \frac{1}{\tilde{\beta}}$. Finally, from the expression of the real eigenvalues, we can directly observe that $\lambda_1 > \lambda_2$ since $\psi_1 > 0$ and $\Delta \neq 0$ $\blacksquare$

Claim 2 *Whether the dynamics are complex or not depends on the following parameter combinations:*

1. *For* $\frac{1}{2} \leq \theta < 1$ *or* $\{0 < \theta < \frac{1}{2}$ *and* $\tilde{\beta} \geqslant 2(1-2\theta)\}$ *the dynamics are non-complex.*

2. *For* $\frac{1}{4} \leq \theta < \frac{1}{2}$ *and* $\tilde{\beta} < 2(1-2\theta)$ *the dynamics are non-complex.*

3. *For* $0 < \theta < \frac{1}{4}$ *and* $\tilde{\beta} + \frac{1}{\tilde{\beta}} > 2(1-2\theta)$ *the dynamics are non-complex.*

4. *For* $0 < \theta < \frac{1}{4}$ *and* $\tilde{\beta} + \frac{1}{\tilde{\beta}} < 2(1-2\theta)$ *the dynamics are complex.*

Proof. The dynamics are (non-)complex iff $\Delta (>) < 0$.

1. For $\frac{1}{2} \leq \theta < 1$ or $\{0 < \theta < \frac{1}{2}$ and $\tilde{\beta} \geqslant 2(1-2\theta)\}$, it is easy to verify that $2(1-2\theta) - \tilde{\beta} < \frac{1}{\tilde{\beta}}$ since $\tilde{\beta} > 0$. Therefore, from Claim 1, $\Delta > 0$.

2. By contradiction. Let us assume, for $\frac{1}{4} \leq \theta < \frac{1}{2}$ and $\tilde{\beta} < 2(1-2\theta)$, that $\Delta < 0$. Claim 1 would then imply that $2(1-2\theta) > \frac{1}{\tilde{\beta}} + \tilde{\beta}$. Therefore, since all terms are non-negative, we conclude that $2(1-2\theta) > \frac{1}{\tilde{\beta}}$ and $2(1-2\theta) > \tilde{\beta}$. This can be rewritten as $2(1-2\theta) > \tilde{\beta} > \frac{1}{2(1-2\theta)}$. The condition holds iff $4(1-2\theta)^2 > 1$. This reduces to $2(1-2\theta) > 1$ since $(1-2\theta) > 0$. Hence $\theta < \frac{1}{4}$, which contradicts our initial assumption on θ.

3. It follows from equation (23).

4. It follows from equation (23).

Finally notice that the case $\tilde{\beta} + \frac{1}{\tilde{\beta}} = 2(1-2\theta)$ is not possible since $\Delta \neq 0$ (see Claim 1) $\blacksquare$

Claim 3 *Considering the definition of* Δ *in Claim 1, we can establish the following results:*

1. *If* $\frac{1}{2} \leq \theta < 1$ *then* $1 - \frac{\sqrt{\Delta}}{2} < 0$.

2. *For non-complex dynamics,* $1 + \frac{\sqrt{\Delta}}{2} > \frac{(1+\psi_1)}{2}$.

3. *If* $\frac{1}{2} \leq \theta < 1$ *then* $\frac{(1+\psi_1)}{2} > 1 - \frac{\sqrt{\Delta}}{2}$.

4. *If* $0 < \theta < \frac{1}{2}$ *and* $\tilde{\beta} \geqslant 2(1-2\theta)$ *then* $\frac{(1+\psi_1)}{2} > 1 - \frac{\sqrt{\Delta}}{2}$.

5. *If* $\frac{1}{4} \leq \theta < \frac{1}{2}$ *and* $\tilde{\beta} < 2(1-2\theta)$ *then* $1 - \frac{\sqrt{\Delta}}{2} > 0$.

6. *If* $\frac{1}{4} \leq \theta < \frac{1}{2}$ *and* $\tilde{\beta} < 2(1-2\theta)$ *then* $\frac{(1+\psi_1)}{2} > 1 - \frac{\sqrt{\Delta}}{2}$.

Proof.

1. Let us first show that $1 - \frac{\sqrt{\Delta}}{2} \neq 0$. Suppose the contrary, $1 = \frac{\sqrt{\Delta}}{2}$. Then $\tilde{\beta} = (1-2\theta) \pm 2\sqrt{2\theta^2 - 3\theta + 1}$. For $\theta = 1/2$, $\tilde{\beta} = 0$ and we find a contradiction because $\tilde{\beta} > 0$. Let us study $1/2 < \theta < 1$. It can be verified that in this case $2\theta^2 - 3\theta + 1 \neq 0$: $2\theta^2 - 3\theta + 1 = 0$ for $\theta = 0$ or $\theta = 1/2$. We can actually proof that $2\theta^2 - 3\theta + 1 < 0$: assuming that $2\theta^2 - 3\theta + 1 > 0$, $3 < \frac{1}{\theta} + 2\theta$; however, for $1/2 < \theta < 1$, it is easy to check that $3 > \frac{1}{\theta} + 2\theta$. Nevertheless, this result implies that $\tilde{\beta}$ is complex, which is a contradiction and, consequently, $1 - \frac{\sqrt{\Delta}}{2} \neq 0$. In fact, we can show that $1 - \frac{\sqrt{\Delta}}{2} < 0$. Let us assume the contrary, $1 - \frac{\sqrt{\Delta}}{2} > 0$. This would imply that $4(1-\theta)^2 - 1 > \tilde{\beta}\left[\tilde{\beta} - 2(1-2\theta)\right]$. However, for $\frac{1}{2} \leq \theta < 1$, the RHS$> 0$ while LHS< 0. So we get a contradiction.

2. By contradiction. Let us assume $1 + \frac{\sqrt{\Delta}}{2} \leq \frac{(1+\psi_1)}{2}$. This is equivalent to $(\psi_1 - 1)^2 \geq \Delta (> 0)$. Since $\Delta = (1+\psi_1)^2 - 4(\psi_1 + \psi_2)$, the previous condition would imply that $0 \geq -4\psi_2$. But this is impossible because $\psi_2 < 0$.

3. From statement 1 of this claim, we already know that $1 - \frac{\sqrt{\Delta}}{2} < 0$ if $\frac{1}{2} \leq \theta < 1$. Our result is then verified since $\psi_1 > 0$.

4. Let us consider, by contradiction, that $\frac{(1+\psi_1)}{2} \leq 1 - \frac{\sqrt{\Delta}}{2}$. This would imply that $\psi_1 - 1 \leq -\sqrt{\Delta}$. We know that $\psi_1 - 1 = \frac{\tilde{\beta} - (1-2\theta)}{1-\theta}$, which is strictly positive since $\tilde{\beta} \geq 2(1-2\theta) > (1-2\theta) > 0$. We then get a contradiction since $-\sqrt{\Delta} < 0$.

5. Let us first show that $1-\frac{\sqrt{\Delta}}{2} \neq 0$. Suppose the contrary, $1 = \frac{\sqrt{\Delta}}{2}$. Then, $\tilde{\beta} = (1-2\theta) \pm 2\sqrt{2\theta^2 - 3\theta + 1}$. For $\frac{1}{4} \leq \theta < \frac{1}{2}$, one can verify that $2\theta^2 - 3\theta + 1 > 0$ and $(1-2\theta) > 0$. We can show that $\tilde{\beta} = (1-2\theta) + 2\sqrt{2\theta^2 - 3\theta + 1}$ is impossible: let us rewrite $\tilde{\beta} = (1-2\theta) + \sqrt{(1-2\theta)^2 + [4(1-\theta)^2 - 1]}$; it is easy to check that $0 < \tilde{\beta} < 2(1-2\theta) < (1-2\theta) + \sqrt{(1-2\theta)^2 + a}$ for $a > 0$; recalling $a = 4(1-\theta)^2 - 1$, since $4(1-\theta)^2 - 1 > 0$ for $0 < \theta < 1/2$, the previous expression provides the contradiction $\tilde{\beta} < \tilde{\beta}$. Similarly, $\tilde{\beta} = (1-2\theta) - 2\sqrt{2\theta^2 - 3\theta + 1}$ cannot hold either: since $\tilde{\beta} > 0$, this would imply that $(1-2\theta) > \sqrt{(1-2\theta)^2 + [4(1-\theta)^2 - 1]}$; however, $(1-2\theta) < \sqrt{(1-2\theta)^2 + a}$ for $a > 0$; as above, recalling $a = 4(1-\theta)^2 - 1 (> 0)$ gives us a contradiction.

   We know now that $1 - \frac{\sqrt{\Delta}}{2} \neq 0$. In order to show that $1 - \frac{\sqrt{\Delta}}{2} > 0$, let us assume $1 - \frac{\sqrt{\Delta}}{2} < 0$. This would imply that $\tilde{\beta}[\tilde{\beta} - 2(1-2\theta)] > 4(1-\theta)^2 - 1$. But this is not possible since the LHS< 0 and the RHS> 0.

6. By contradiction. We assume $\frac{(1+\psi_1)}{2} \leq 1 - \frac{\sqrt{\Delta}}{2}$, which would imply $(\psi_1 - 1) \leq -\sqrt{\Delta}$. On the one hand, $-\sqrt{\Delta} < 0$ because we are in a situation of non-complex dynamics. On the other hand, $(\psi_1 - 1) = \frac{\tilde{\beta} - (1-2\theta)}{1-\theta}$, where we can identify two cases for $\tilde{\beta} < 2(1-2\theta)$: (i) $(1-2\theta) \leq \tilde{\beta} < 2(1-2\theta)$ and (ii) $\tilde{\beta} < (1-2\theta) < 2(1-2\theta)$. Case (i) yields a contradiction since $(\psi_1 - 1) > 0$. Case (ii) would imply $(\psi_1 - 1)^2 \geq \Delta$, which is impossible for non-complex dynamics (see proof of Claim 3, statement 2).

■

6 References

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